

The Spirit of Unification: The Wei of Physics*

Orfeu Bertolami[†]

Departamento de Física, Instituto Superior Técnico

Avenida Rovisco Pais 1, 1049-001 Lisboa, Portugal

Abstract

We review, in a historical perspective, developments in physics which led to the emergence of unifying ideas and theories. Some attention is paid to the theoretical programme that started in the second decade of the XXth century and whose objective was to reach a unified description of gravity and electromagnetism. These attempts can be regarded as conceptually akin to the contemporary “theories of everything” which aim to unify all interactions of Nature.

* Talk delivered at the “Mira Fernandes and his age: An historical Conference in honour of Aureliano de Mira Fernandes (1884-1958)”, 17 June 2009, Instituto Superior Técnico, Lisbon, Portugal.

[†] Also at Instituto de Plasmas e Fusão Nuclear, IST. E-mail: orfeu@cosmos.ist.utl.pt

I. INTRODUCTION

Unifying ideas in science and philosophy: a chronology

Many philosophical systems aimed at unifying the knowledge of their time about the physical world. In physics this trend has been more visible from the second decade of the XXth century onwards, when the first attempts to unify gravity and electromagnetism were developed. However, one should realize that the search for unifying principles is at the very core of the development of physics since early on.

In what follows we present a brief chronology of some of the most relevant unifying ideas in physics and philosophy:

ca. 440 BC: The Pre-Socratics Greek philosophers Leucippus and Democritus were the first to put forward the idea that *everything is composed of atoms* and that *atoms are physically indivisible and in constant motion in empty space*. This is a quite elegant solution to the problem of maintenance of identity in a world in constant transformation.

384-322 BC: Aristotle's *Physics*, where *physis* means Nature, treats the problem of growth and change in the whole natural world, which includes plants, animals and objects. Aristotle also unifies motion and movement, which manifest themselves through the change in quality and in quantity.

ca. 630 AD: Islam arises as a religious movement that claims to unify the Abrahamic monotheistic religions.

ca. 1650: Emergence of the scientific revolution that laid the basis of Newton's mechanics and the law of universal gravitation, whose applicability everywhere in the Universe made possible systematizing the knowledge of the whole physical world.

ca. 1850: French philosopher Auguste Comte put forward his Positivist Philosophy as, in his view, humankind was living in the scientific phase of its history and as such all sciences, natural and human, should be regarded as a whole.

ca. 1870: Karl Marx's historical materialism aimed to understand the whole human history through the economical relationships present in society.

1859: Charles Darwin (and Alfred Russell Wallace) put forward the theory of evolution and the precept of survival of the fittest, which should apply to all living organisms. Dar-

win's theory has influenced many thinkers, and his ideas were often improperly used, most particularly in social sciences. A few decades later, Ludwig Boltzmann outlined his idea of *theoretical pluralism*, according to which scientific theories should be regarded as representations of Nature that should “compete for their survival”, like living organisms. According to Boltzmann, the ultimate scientific knowledge was unattainable — a remarkably modern view, quite akin to Popper's view that scientific theories are actually no more than conjectures that remain valid as far as they account for the observational facts.

1865: Maxwell's equations unify electricity, magnetism, optics and electromagnetism.

1869: Mendeleyev's periodic system explains all chemical bounds through the common properties of elements (92 natural ones) properly gathered according to the number of electrons available for a chemical reaction.

The Relativity Revolution

1905: Einstein's Special Relativity (SR) unifies electromagnetism and a generalized mechanics, an approach that allows for the whole physics to be expressed in a frame independent way and invariant under the Lorentz transformations.

1908: Minkowski shows that SR is more meaningful with a further unification of space and time so to treat physical phenomena in a space-time continuum (see Refs. [1, 2] for thorough discussions).

1915: Einstein's General Relativity unifies SR, gravity and Riemannian differential geometry imposing that physics should be invariant under general coordinate transformations, *i.e.*, diffeomorphisms. This is achieved by promoting the space-time metric, $g_{\mu\nu}$, which determines distances in space-time

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu , \quad (1)$$

into a dynamical variable that solves the so-called Einstein's field equations for a given matter-energy configuration described by the energy-momentum tensor $T_{\mu\nu}$:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} , \quad (2)$$

where $R_{\mu\nu}$ is the Ricci tensor, obtained by a contraction of the Riemann tensor, the fundamental tensor of differential geometry to detect curvature, $R = R^\lambda_\lambda$, G is Newton's constant and c the speed of light.

General Relativity assumes that space-times admits a symmetric and metric compatible connection, $g_{\mu\nu;\lambda} = 0$, the Levi-Civita connection. The Riemann tensor is built with this connection and the contracted version of the Bianchi identity implies that the energy-momentum tensor is covariantly conserved. This formulation emphatically states that physics is a science of space-time (see Ref. [3] for an extensive discussion). Furthermore, the theory accounts for all known observational facts, from the solar system to the largest observed scales — provided that in the latter, dark energy and dark matter are included into the cosmological description (see Ref. [4] for a thorough discussion).

Somewhat later, in 1917, while discussing for the first time the cosmological implications of GR, Einstein [5] rejected a legitimate solution of his equation that predicted the expansion of the Universe and, rather unfortunately, concluded that the field equations were incomplete and should be implemented with a constant term, the cosmological constant, Λ :

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} . \quad (3)$$

One could assume that the observed expansion of the Universe would render the cosmological constant redundant. However, the recently detected acceleration on the expansion of the Universe [6] brings back the cosmological constant, which suitably adjusted, is the simplest possible explanation for this observational fact.

Unitary Field Theories¹

1918: Weyl's dilatation unitary field theory (see Refs. [7, 8] for thorough discussions).

Weyl admits that the metric is not compatible, which is expressed via a scalar field, $\lambda(x)$, assuming the following properties

$$g_{\mu\nu;\lambda} := Q_{\mu\nu\lambda} \neq 0 \quad (4)$$

$$\bar{g}_{\mu\nu} = \lambda(x)g_{\mu\nu} \quad , \quad \lambda(x) \in C^\infty(R) \quad (5)$$

$$Q_{\mu\nu}^\lambda = -g_{\mu\nu}Q^\lambda \quad , \quad Q = Q_\lambda dx^\lambda \quad (6)$$

$$Q_\lambda \rightarrow \bar{Q}_\lambda + \frac{\partial \ln \lambda(x)}{\partial x^\lambda} \quad (7)$$

$$dQ = \left(\frac{\partial Q_\lambda}{\partial x^\rho} - \frac{\partial Q_\rho}{\partial x^\lambda} \right) dx^\rho \wedge dx^\lambda := F_{\rho\lambda} dx^\rho \wedge dx^\lambda \quad (8)$$

¹ The word unitary is used in this section for historical reasons. It means unified and not unitary in the mathematical sense.

$$V_{;\nu}^\mu = V_{,\nu}^\mu + \Gamma_{\nu\rho}^\mu V^\rho \quad , \quad \omega_{\mu;\nu} = \omega_{\mu,\nu} + \Gamma_{\nu\mu}^{\prime\rho} \omega_\rho \quad (9)$$

$$\Gamma_{\nu\rho}^{\prime\lambda} = -\Gamma_{\nu\rho}^\lambda \quad (10)$$

and then imposes the identification with the electromagnetic field $Q_\rho \leftrightarrow A_\rho$.

His generalized Riemann curvature is built from his new connection

$$K_{\rho[\mu\nu]}^\lambda := \partial_\mu \Gamma_{\nu\rho}^\lambda + \Gamma_{\mu\theta}^\lambda \Gamma_{\nu\rho}^\theta - \partial_\nu \Gamma_{\mu\rho}^\lambda - \Gamma_{\nu\theta}^\lambda \Gamma_{\mu\rho}^\theta \quad , \quad (11)$$

yielding the Ricci tensor

$$K_{\mu\nu\lambda}^\lambda := \partial_\mu \Gamma_{\nu\lambda}^\lambda + \Gamma_{\mu\theta}^\lambda \Gamma_{\nu\lambda}^\theta - \partial_\nu \Gamma_{\mu\lambda}^\lambda - \Gamma_{\nu\theta}^\lambda \Gamma_{\mu\lambda}^\theta \quad (12)$$

and the scalar curvature:

$$K(g, \Gamma) = g^{\mu\nu} K_{\mu\nu} \quad , \quad K_{\mu\nu} := K_{\mu\nu\lambda}^\lambda \quad . \quad (13)$$

One could also consider theories with a non-symmetric connection, so that *torsion* is non-vanishing:

$$S_{\mu\nu}^\lambda := \Gamma_{[\mu\nu]}^\lambda \quad , \quad S_\mu = S_{\mu\rho}^\rho \quad , \quad (14)$$

and the following identity among the relevant quantities is satisfied:

$$V_{\mu\nu} + K_{[\mu\nu]} = -4S_{[\mu;\nu]} + 2S_{\mu\nu;\lambda}^\lambda + 4S_{\mu\nu}^\lambda S_\lambda \quad . \quad (15)$$

Clearly, if the connection is symmetric

$$\Gamma_{\mu\nu}^\lambda = \Gamma_{(\mu\nu)}^\lambda \leftrightarrow S_{\mu\nu}^\lambda = V_{\mu\nu} = K_{[\mu\nu]} = 0 \quad , \quad (16)$$

and one recovers the Levi-Civita connection which can be expressed in terms of the Christoffel symbols:

$$\Gamma_{\mu\nu}^\lambda = \left\{ \begin{matrix} \lambda \\ \mu\nu \end{matrix} \right\} := \frac{1}{2} g^{\lambda\sigma} (h_{\mu\sigma,\nu} + h_{\nu\sigma,\mu} - h_{\mu\nu,\sigma}) \quad , \quad (17)$$

where one has written the most general metric-like tensor as $g_{\mu\nu} = h_{\mu\nu} + k_{\mu\nu}$, where $h_{\mu\nu} := g_{(\mu\nu)} := \frac{1}{2}(g_{\mu\nu} + g_{\nu\mu})$ and $k_{\mu\nu} := g_{[\mu\nu]} := \frac{1}{2}(g_{\mu\nu} - g_{\nu\mu})$.

1919, 1925: Kaluza-Klein approach

The Finish physicist Gunnar Nordström was the first to speculate, in 1909, that space-time could have more than four dimensions. A concrete construction based on this idea was

put forward by Theodor Kaluza in 1919, who showed that an unified theory of gravity and electromagnetism could be achieved through a 5-dimensional version of general relativity, provided the extra dimension was compact and fairly small, and could therefore have passed undetected. This approach was reexamined by Oskar Klein in 1925, who showed that after correcting some of Kaluza's assumptions, the resulting 4-dimensional theory contained general relativity, electromagnetism and a scalar field theory.

This approach was very dear to Einstein till late in his life [9], and has been followed in the most important attempts to unify all known four interactions of Nature. In a more modern language, the basic underlying feature of the Kaluza-Klein approach is the assumption that the Universe is described by Einstein's general relativity in a 5-dimensional Riemannian manifold, M_5 , that is a product of a 4-dimensional Riemannian manifold, M_4 , our world, and tiny one-dimensional sphere, S_1 .

Much later, in the 1970's, it was shown the impossibility of encompassing larger gauge groups than $U(1)$ in 4 dimensions starting from D -dimensional general relativity. Furthermore, besides the impossibility of this "monistic" approach, *i.e.*, the route from higher-dimensional gravity down to 4-dimensional gravity plus gauge theories, some "no-go theorems" have shown that no non-trivial gauge field configurations could be obtained in 4 dimensions from D -dimensional gravity theory without higher-curvature terms (see *e.g.* Ref. [10] for the case of $N = 1$ supergravity in $D = d + 4 = 10$ dimensions). Another quite relevant result is that, in order to obtain chiral fermions in our 4-dimensional world, D must be even if all extra dimensions are compact [11].

Actually, in order to get consistent effective 4-dimensional models arising for instance from a D -dimensional Einstein-Yang-Mills theory, multidimensional universes of the form $M_D = \mathbf{R} \times G^{\text{ext}}/H^{\text{ext}} \times G^{\text{int}}/H^{\text{int}}$, should be considered, where $G^{\text{ext(int)}}$ and $H^{\text{ext(int)}}$ are respectively the isometry groups in $3(d)$ dimensions. This technique, known as *coset space dimensional reduction* [12] (see Ref. [13] for an extensive discussion), is quite powerful and has many applications in theoretical physics. For example, in cosmology, when considering homogeneous and isotropic models (a 1-dimensional problem), it allows for obtaining consistent effective models arising from 4-dimensional [14] or D -dimensional Einstein-Yang-Mills-Higgs theories [15]. In this particular instance, one considers $G^{\text{ext(int)}} = SO(4)$ ($SO(d+1)$) and $H^{\text{ext(int)}} = SO(3)$ ($SO(d)$) as the homogeneity and isotropy isometry groups in $3(d)$ dimensions.

We shall further discuss the more recent efforts based on the Kaluza-Klein approach

below. Let us now get back to the historical discussion of the unitary field theories.

1921: Eddington's Affine Geometry approach assumes a symmetric connection:

$$\Gamma_{\mu\nu}^{\lambda} = \Gamma_{(\mu\nu)}^{\lambda} \quad (18)$$

so that for a constant, λ ,

$$g_{\mu\nu} = \lambda K_{(\mu\nu)} \quad , \quad K_{(\mu\nu)} = K_{\mu\nu\lambda}^{\lambda} \quad (19)$$

together with a metric compatibility:

$$\nabla_{\mu} K_{(\nu\lambda)}(\Gamma) = 0 \quad . \quad (20)$$

Electromagnetism, that is, Maxwell's non-homogeneous Eqs., $F_{;\nu}^{\mu\nu} = J^{\mu}$, arise from the identification of the gauge field strength:

$$F^{\mu\nu} := g^{\mu\lambda}(\Gamma) g^{\nu\rho}(\Gamma) K_{[\lambda\rho]}(\Gamma) \quad (21)$$

$$F_{\mu\nu} = \partial_{\mu}\Gamma_{\nu} - \partial_{\nu}\Gamma_{\mu} \quad , \quad (22)$$

with a generalized Ricci tensor:

$$K_{\mu\nu}(\Gamma) = R_{\mu\nu}(\Gamma) + F_{\mu\nu}(\Gamma) \quad . \quad (23)$$

The conservation of charge $J_{;\mu}^{\mu} = 0$ is achieved through the condition (cf. Eq. (14)):

$$S_{\mu\rho}^{\lambda} = 0 \quad . \quad (24)$$

In 1923, Einstein proposed an action for Eddington's approach:

$$\mathcal{L} = 2\sqrt{-\det(K_{\mu\nu})} \quad , \quad (25)$$

with

$$\lambda^2 K_{\mu\nu} = g_{\mu\nu} + \phi_{\mu\nu} \quad (26)$$

where $g_{\mu\nu} = g_{(\mu\nu)}$, $\phi_{\mu\nu} = \phi_{[\mu\nu]}$ and

$$\phi_{\mu\nu} := \frac{1}{2} \left(\frac{\partial \Gamma_{\mu\lambda}^{\lambda}}{\partial x^{\nu}} - \frac{\partial \Gamma_{\nu\lambda}^{\lambda}}{\partial x^{\mu}} \right) \quad . \quad (27)$$

It is rather interesting that the relevant quantities are obtained through a variation with respect to the contracted connection, $\Gamma_{\mu\lambda}^{\lambda}$. Approaches where metric and connection are

regarded independently in the variational problem are usually referred to as *first order formalism*. Einstein assumes that the electromagnetic field strength and current are given respectively by:

$$f^{\mu\nu} := \frac{\delta \mathcal{L}}{\delta \phi_{\mu\nu}} \quad , \quad j^\mu := \frac{\partial \hat{f}^{\mu\nu}}{\partial x^\nu} . \quad (28)$$

In the same year, there followed another proposal by Einstein in order to get a more encompassing approach, which reflects somehow a sign of his dissatisfaction with the whole procedure [7, 8].

1922-1923, 1928: Independently, Cartan and Einstein developed the distant parallelism approach, in which torsion plays a central role. One introduces the tetrad or *vierbein* field, e^μ_α , that relates the flat tangent space to the Riemannian space metric through the relationship:

$$g^{\mu\nu} = e^\mu_\alpha e^\nu_\beta \eta^{\alpha\beta} , \quad (29)$$

so that

$$e^\lambda_{\alpha;\mu} = 0 . \quad (30)$$

The connection is given by

$$\Gamma^\lambda_{\mu\nu} = e^\lambda_\alpha \frac{\partial e^\alpha_\nu}{\partial x^\mu} , \quad (31)$$

and torsion by the antisymmetric combination

$$S^\alpha_{\mu\nu} = e^\lambda_\alpha \partial_{[\mu} e^\alpha_{\nu]} . \quad (32)$$

The proposed Lagrangian density is the following

$$\mathcal{L} = \varrho g^{\mu\nu} S^\lambda_{\mu\rho} S^\rho_{\nu\lambda} \quad , \quad \varrho = \det(e^\lambda_\alpha) = \sqrt{-\det(g_{\mu\nu})} , \quad (33)$$

whose physical meaning is elusive.

Despite their limited success, these ideas allowed Cartan to obtain a set of equations, today referred to as Maurer-Cartan structural equations, which are nowadays used as the standard technique to obtain from the basis of 1-forms, $[\omega^\nu]$, dual to the tetrad, the torsion 1-form, S^μ , and curvature 2-form, R^μ_ν , for a generic connection that has symmetric as well as antisymmetric components:

$$d\omega^\mu + \theta^\mu_\nu \wedge \omega^\nu = S^\mu \quad , \quad S^\mu := \frac{1}{2} T^\mu_{\nu\lambda} \omega^\nu \wedge \omega^\lambda \quad (34)$$

$$R^\mu_\nu = d\theta^\mu_\nu + \theta^\mu_\lambda \wedge \theta^\lambda_\nu \quad , \quad R^\mu_\nu := \frac{1}{2} R^\mu_{\nu\lambda\rho} \omega^\lambda \wedge \omega^\rho , \quad (35)$$

where one has introduced the connection 1-form, θ_ν^μ :

$$\theta_\nu^\mu := \Gamma_{\lambda\nu}^\mu \omega^\lambda, \quad (36)$$

and used the well known antisymmetric wedge product.

1931-1932: The Italian mathematician Paolo Straneo from Genova considered a generalized connection, so that different contractions of the resulting Riemann curvature would concern gravity and electromagnetism, respectively. His set up is the following:

$$\Gamma_{\mu\nu}^{\prime\lambda} = -\Gamma_{\mu\nu}^\lambda, \quad \Gamma_{\mu\nu}^\lambda = \left\{ \begin{smallmatrix} \lambda \\ \mu\nu \end{smallmatrix} \right\} + \Omega_{\mu\nu}^\lambda, \quad (37)$$

which yield the following field equations:

$$K_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + 2\left(\frac{\partial\psi_\mu}{\partial x^\nu} - \frac{\partial\psi_\nu}{\partial x^\mu}\right) = T_{\mu\nu}, \quad (38)$$

where ψ_μ is the electromagnetic gauge field. The Riemann tensor is found to be

$$R_{\mu\nu\rho}^\lambda + \Omega_{\mu\rho;\nu}^\lambda + \Omega_{\mu\rho}^\alpha \Omega_{\alpha\nu}^\lambda = 0 \quad (39)$$

leading, through contraction, to gravity

$$R_{\mu\nu} + \Omega_{\rho\mu}^\lambda \Omega_{\lambda\nu}^\rho = 0 \quad (40)$$

and electromagnetism

$$\Omega_{\mu\rho;\lambda}^\lambda = 0. \quad (41)$$

Hence, the issue is the choice of the connection generalization that allows for the desired result.

The contribution of Mira Fernandes to the Unitary Field Theory Discussion

Let us now focus on the contribution of Aureliano Mira Fernandes of Instituto Superior Técnico, in Lisbon. His proposal appeared in two notes on the Italian journal *Rendiconti della Real Accademia dei Lincei*, in 1932 [16] and 1933 [17] and relies on the work of the Italian mathematician Straneo, as discussed above. He considered three possibilities. In his first approach he introduced two new fields:

$$C_{\mu\nu}^\lambda = \Gamma_{\mu\nu}^\lambda + \Gamma_{\mu\nu}^{\prime\lambda} = C_\mu A_\nu^\lambda \quad (42)$$

$$(43)$$

so that

$$\Gamma_{\mu\nu}^\lambda = \left\{ \begin{smallmatrix} \lambda \\ \mu\nu \end{smallmatrix} \right\} + C_\nu A_\mu^\lambda \quad , \quad \Gamma_{\mu\nu}'^\lambda = - \left\{ \begin{smallmatrix} \lambda \\ \mu\nu \end{smallmatrix} \right\} \quad , \quad (44)$$

which lead to Straneo's field equations through the identification with the electromagnetic field, $-2\psi_\mu = C_\mu$, assuming that

$$S_{\mu\nu}^\lambda := \frac{1}{2} (\Gamma_{\mu\nu}^\lambda - \Gamma_{\nu\mu}^\lambda) = 0 \quad , \quad S_{\mu\nu}'^\lambda := \frac{1}{2} (\Gamma_{\mu\nu}'^\lambda - \Gamma_{\nu\mu}'^\lambda) = 0 \quad . \quad (45)$$

The above assumption is also considered in his second approach, where $C_{\mu\nu}^\lambda = C_\mu A_\nu^\lambda$, and:

$$\Gamma_{\mu\nu}^\lambda = \left\{ \begin{smallmatrix} \lambda \\ \mu\nu \end{smallmatrix} \right\} \quad , \quad \Gamma_{\mu\nu}'^\lambda = - \left\{ \begin{smallmatrix} \lambda \\ \mu\nu \end{smallmatrix} \right\} + C_\nu A_\mu^\lambda \quad , \quad (46)$$

which allows for obtaining Straneo's field equations through the identification with the electromagnetic field, $2\psi_\mu = C_\mu$.

Finally, a third choice involves the combination of the two previous ones:

$$\Gamma_{\mu\nu}^\lambda = \left\{ \begin{smallmatrix} \lambda \\ \mu\nu \end{smallmatrix} \right\} + \Omega_{\mu\nu}^\lambda \quad , \quad \Gamma_{\mu\nu}'^\lambda = - \left\{ \begin{smallmatrix} \lambda \\ \mu\nu \end{smallmatrix} \right\} + \Omega_{\mu\nu}'^\lambda \quad (47)$$

so that

$$C_{\mu\nu}^\lambda = \Omega_{\mu\nu}^\lambda + \Omega_{\mu\nu}'^\lambda \quad , \quad (48)$$

which allows for satisfying Straneo's field equations if each of the curvatures resulting from the above connections are taken to vanish separately.

At this point it is interesting to mention Aureliano Mira Fernandes' thoughts on the unification principle. These appeared in a book published in 1933, based on lectures he presented at the Instituto de Altos Estudos of the Academia das Ciências de Lisboa in February 1st and 4th, 1933, "Modernas Concepções da Mecânica" [18]:

"One has then reached Riemann's vision according to which physical phenomena should be governed by an underlying geometrical concept; and conversely, that geometry is determined by the content of space. The unity principle, as a scientific and philosophical goal, is philosophically boosted and dignified. And in the process, domains of mathematical thinking get better equipped for future requirements of the physical theories. ... This is a singular fate for a science whose evolution becomes, by rule, more interesting as more detached it gets from the realm of the direct applications!"²

² "Atingiu-se o ideal de Riemann da subordinação de todos os fenómenos físicos a uma concepção

Let us take on from this quote and reflect a bit upon the unification principle and add up some of our own thoughts on the matter. Unification seems to be a natural trend in science as, from time to time, knowledge acquired about a class of natural phenomena allows for unifying theories and models that explain apparently distinct phenomena. Unification is achieved in the sense that qualitatively distinct phenomenological manifestations can be explained by a common underlying theory or model. This qualitative change is possible if, and only if, broad and detailed experimental data are available. Furthermore, the unification of theories and models leads to conceptual simplicity, even though most often at expense of mathematical complexity.

It is important to point out that unification is more than a methodological procedure, as the unity of Nature imposes that all its phenomenological manifestations should be regarded as a whole, without artificial divisions, and hence have a correspondence with particular elements of any serious theoretical framework that aims at describing the most salient features of the Universe. From this perspective, and given the lack of decisive phenomenological facts to guide their efforts, it is not surprising that the enterprise carried by Einstein, Weyl, Eddington, Cartan and followers could not be successful in its aim to unify gravity and electromagnetism. One could add that their disregard of Quantum Mechanics and its methods to address the nuclear and subnuclear interactions was also a highly questionable methodological choice.

Despite these shortcomings, on a broad sense, their contributions were nevertheless valuable — as they threaded their way through unknown theoretical landscapes, mapped the encountered fruitless regions and, what is perhaps more relevant, devised methods that have been useful in many areas of physics. Their failure also illustrates that a purely theoretical approach is unlikely to achieve the unification goal through the labyrinth of theoretical and mathematically valid possibilities. Phenomenological guidelines are essential and cannot be disregarded in the process of building new theories and models. For sure, phenomenology on its own is no more than an inert compilation of data, notwithstanding the recent emergence of increasingly powerful computers and huge databases, which have

geométrica; e inversamente, da determinação da geometria pelo conteúdo espacial. O princípio da unidade, como ideal científico e filosófico fica filosoficamente robustecido e dignificado. E ficam também, agora como sempre, os domínios do pensamento matemático sobejamente apetrechados para uma possível interpretação de futuras exigências das teorias físicas. ... Singular destino este, o duma ciência cuja evolução é, em regra, tanto mais interessante quanto mais desinteressada!”.

led to the idea that models and theories can be built algorithmically — most particularly in information and social sciences, and in engineering and management [19]. This author is skeptical whether these methods can be straightforwardly applied to physics, given that, most often, physical theories go much beyond the phenomenology that is available at the time they are proposed. It is precisely for these reasons that we believe that the spirit of unification works as a sort of taoist *Wei* principle, that is to say, an intuition, a feeling about “acting or not acting”, about “doing, but not overdoing” in the process of building new unified physical theories.

The Quantum Revolution

Let us now turn to another fundamental development in XX’s century physics. Somewhat after the development of General Relativity, the understanding of microphysics (atoms, molecules, nuclear physics and beyond) made possible the emergence of Quantum Mechanics (QM). The theory has gone itself through a process of methodological unification from 1924 till the 1930’s. Indeed, the set of principles that were put forward by Bohr and Heisenberg (The Matter-Wave Complementarity Principle and The Uncertainty Principle) have provided abundant insight about the need to pursue a fresh new vision of Nature in what concerned microphysical phenomena. Two distinct formulations were then put forward: matrix mechanics, proposed by Heisenberg, Born and Jordan in 1925; and, in 1926, wave mechanics, based on Schrödinger’s wave equation. These two formulations were shown to be unitarily equivalent, a result known as the Stone-von Neumann theorem.

In 1928, Dirac presented the relativistic version of the wave equation, and showed that its solutions admit particles and *antiparticles*, states with the same mass, but with opposite charge and baryon (lepton) number. In 1932, the discovery of the positron (the anti-electron) by Carl Anderson did confirm the physical validity of Dirac’s equation, which unifies QM with SR and the concept of spin.

Actually, somewhat earlier, in 1926, Dirac suggested that analytical classical mechanics was connected with QM through a formal relationship between Poisson’s brackets and the commutator of physical observables:

$$\{ \ , \ }_{\text{PB}} \rightarrow \frac{1}{i\hbar} [\ , \] . \quad (49)$$

This connection requires the introduction of Planck’s constant, a distinct feature of QM, and the imaginary unit, which arises as the fundamental entity of QM: the wavefunction

that solves Schrödinger's wave equation is not an observable, being hence undetermined up to an imaginary phase (see *e.g.* Ref. [20] for an extremely elegant presentation).

In 1932, Wigner introduced the so-called quasi-probability distribution, while studying the quantum corrections to classical statistical mechanics. This distribution allows to link Schrödinger's wave function to a probability distribution in phase space, as it maps real phase-space functions to Hermitian operators. Thus, a formal unification of QM with statistical mechanics is achieved. Later, in 1949, Moyal did show that this distribution is the basis of the encoding of all quantum expectation values, and hence quantum mechanics in phase space. These developments are the basis of the most recent phase-space noncommutative generalizations of QM [21–23].

Finally, in 1948, Feynman has developed his path integral formalism and shown its equivalence with QM.

Other relevant developments towards an unified view of Nature include:

1934: Fermi's theory of the weak force describes the interactions through the contact of current densities composed by two fermions each, $J_\mu = \bar{\psi}\gamma_\mu\psi$, with strength $G_F \simeq 10^{-5} m_p^{-2}$, where m_p is the proton mass and γ_μ denotes the Dirac's matrices:

$$\mathcal{L}_W = G_F J_\mu J^\mu . \quad (50)$$

Fermi's theory is an important precursor of the field theory models that would eventually describe the fundamental interactions.

Quantum Electrodynamics (QED): In late 1940's and early 1950's, Feynman, Schwinger, Tomonaga, Dyson and others have developed the basic tools of quantum field theory and have shown that QED, the quantum field theory of charges and photons, was a fully consistent theory that could achieve, through the renormalization procedure, an impressive power of predictability (see *e.g.* Ref. [24] for an insightful account).

Yang-Mills Theory: In 1954, Yang and Mills (and independently, Salam and Shaw) did propose a generalization of Maxwell's electromagnetism, based on the $U_{EM}(1)$ gauge group, that was invariant under a $G = SU(2)$ non-abelian gauge group transformation. The resulting Yang-Mills equations are similar to Maxwell's ones and are expressed in terms of the field strength, $F_{\mu\nu}^a$, of the gauge field, A_μ^a :

$$\partial^\mu F_{\mu\nu}^a = J_\nu^a \quad , \quad F_{[\mu\nu,\lambda]}^a = 0 \quad , \quad (51)$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf_{bc}^a A_\mu^b A_\nu^c \quad (52)$$

where a is an internal space index that corresponds to the generators of the gauge group (for the $SU(2)$, $a = 1, 2, 3$), f_{bc}^a are the structure constants of the gauge group, g is the coupling constant, and J_ν^a are the current densities of matter, as suggested by Fermi.

Let us close this subsection by stepping out of physics for a while.

1956: Crick and Watson's discovery of the DNA molecular structure made possible the understanding of the hereditary mechanism and of the basic features of the protein synthesis of all living organisms. This revolution has led to a remarkable progress in biology, the most recent being the Human Genome Project — concluded in 2003, and carried out by teams led by Craig Venter and James Watson. The subsequent sequencing of the genome of other organisms did open the perspective of yet another revolution, through the detailed understanding of the protein synthesis and the possibility of unraveling an unifying paradigm. Actually, the emergence of a unified biology has been discussed for quite some time [25].

1960's: Plate tectonics theory enabled, through the work of Wegener, Holmes, Hess, Deitz and others, to understand geological phenomena on Earth (and on any rocky planet!) in an encompassing way [26].

II. FORCES OF THE WORLD, UNITE!

The Gauge Principle

Electroweak (EW) unification: The unification of the electromagnetic and weak interactions was achieved through the work of Glashow (1961), Salam (1968), Weinberg (1967) and others. The main ingredients include the gauge group $G = SU_L(2) \otimes U_Y(1)$ that breaks down, through the Higgs mechanism, to Maxwell's theory $U_{EM}(1)$. The fundamental *spontaneous symmetry breaking mechanism* that endows the Higgs field with a non-vanishing vacuum expectation value (experimentally, $\langle 0|H|0 \rangle = 246 \text{ GeV}$) is on its own a very interesting example of the unity of Nature, as it is the very process underlying phase transitions in condensed matter physics, the Higgs field vacuum expectation value playing the role of an order parameter, the Higgs field effective potential being equivalent to Helmholtz's free energy.

Quantum Chromodynamics (QCD): In the 1960-1970's, it has been shown that strong

interactions could also be described by a gauge theory, QCD, with gauge group $SU_c(3)$, where c stands for the strong charge dubbed “colour”. This was achieved through the understanding that hadrons could be sorted into groups having similar properties and masses: the “eightfold way”, put forward by Gell-Mann and Ne’eman in 1961. Shortly after, Gell-Mann and Zweig proposed that the strong interaction group structure should be understood through the existence of three distinct “colours” of smaller particles inside the hadrons, the *quarks*. Thus, the basic components of hadrons and mesons are “coloured” quarks and *gluons*, the vector bosons of strong interactions, with dynamics ruled by QCD. The theory has remarkable properties such as *confinement*, which means that coloured states cannot be directly observed, and *asymptotic freedom*, discovered by David Gross, David Politzer and Frank Wilczek in 1973, which allows for predictions of many high energy experiments using the perturbation techniques of quantum field theory. Confinement and asymptotic freedom mean that quarks are strongly coupled at low energies and that their coupling gets logarithmically weaker with the growth of energy, respectively.

The description of the electroweak and the strong interactions through the gauge principle gave rise to the so-called *Standard Model* (SM) of particle interactions, which is so far consistent with most of the particle physics phenomenology. However, there exists evidence that physics beyond the SM is needed to account for the fact that neutrinos seem to be massive and that phenomenology requires a new “sterile” neutrino (see *e.g.* Ref. [27] for an updated discussion). Apart from that, the Higgs boson is the only SM state yet to be detected. The search for the Higgs boson field is the main priority of the Large Hadron Collider (LHC).

Grand Unified Theories

The gauge principle and the SM suggest a natural procedure to build more encompassing models. One considers non-abelian gauge theories with symmetry groups that admit the SM gauge group as a subgroup. These Grand Unified Theories (GUTs) have been extensively discussed from 1973 onwards by Pati, Salam, Georgi, Glashow, Quinn, Weinberg and others (see Ref. [28] for a thorough discussion), and the most studied cases admitted the following GUT gauge groups and symmetry breaking pattern:

$$GUT := SU(5), SO(10), E(6), \dots \rightarrow G_{SM} = SU_c(3) \otimes SU_L(2) \otimes U_Y(1) \rightarrow U_{EM}(1) . \quad (53)$$

It is important to realize that GUTs are also suggested by the evolution of the coupling

constants with energy according to the *renormalization group equations*. Through these equations it is possible to show that if there is no intermediate physics between the EW unification and the GUT, the “great desert” hypothesis, for the $SU(5)$ GUT [29] one finds that the unification scale can be as large as, $E_{GUT} \simeq 10^{17} \text{ GeV}$ [30]. Subsequently, precision data arising from LEP collider, the Z “factory” that ran at CERN from 1989 to 2000, have revealed that the coupling constants of the electromagnetic, strong and weak interactions would meet at about 10^{16} GeV , and hence be consistent with a putative unification, only in the context of the so-called minimal supersymmetric (see below) extensions of the SM [31]. This is a further evidence of physics beyond the SM.

A distinct feature of GUTs is the existence of leptoquark particles whose interactions can mediate violation of baryon and lepton numbers. Violation of baryon number, violation of C and CP discrete symmetries, and out-of-equilibrium processes, which is a natural feature in an expanding Universe, do allow for the creation of the baryon asymmetry of the Universe (BAU). The generation of this asymmetry, usually referred to as *baryogenesis*, is vital to ensure that the Universe does not end up being composed only by photons, as argued by Sakharov in 1967 (see Ref. [32] for a review). An alternative scenario to achieve the BAU involves violation of the baryon number and of the CPT symmetry [33], which might occur in the context of string theory (see below).

The cosmological constant problem

As discussed above, the spontaneous symmetry mechanism is an essential ingredient of the electroweak unification. However, in the process of the Higgs field acquiring a non-vanishing expectation value, the vacuum energy becomes non-vanishing and a cosmological constant proportional to $\langle 0|H|0\rangle^4$ is generated. In quantum field theory, this energy can be disregarded, as only energy differences matter; however, as first pointed out by Zel’dovich in 1968 [34], in a realistic setup the vacuum energy gravitates and cannot be neglected, as it curves space-time (see also Ref. [35] for an updated discussion). The problem is that, even in the most conservative scenario, the SM without any consideration about GUTs, the generated cosmological constant is about 10^{56} orders of magnitude greater than the value inferred from cosmology, namely 10^{-12} eV^4 , as pointed out by Linde [36], Dreitlein [37] and Veltman [36] in 1974-1975. The discrepancy is of $O(10^{108})$ if one considers GUTs, and as huge as 10^{120} in quantum gravity approaches, assuming in this context that the contribution to the vacuum energy density is $O(M_P^4)$, where $M_P = \sqrt{\hbar c/G} = 1.2 \times 10^{19} \text{ GeV}$ is the Planck

mass, the typical scale of quantum gravity.

Thus, in order to make sense of the current description of the Universe, a cancellation of these vacuum contributions must be carried out by terms introduced by hand in the geometrical side of Einstein's field equations, Eq. (3), with the corresponding number of decimal places. Many solutions for this absurd adjustment problem have been proposed (see Refs. [35, 39–41] for discussions): for instance, a suppression of 10^{-120} can be achieved if the vacuum energy density evolves with cosmic time, t , as $\rho_V \sim t^{-2}$ [42].

The cosmological constant problem is a major gap in our understanding of the unity of Nature, as it indicates that the quantum field description of the microscopical world does not match the general relativistic description of the Universe. It is a riddle that any fundamental theory must address. Unfortunately, string theory, the most studied quantum gravity approach, has not provided a decisive insight on the nature of a possible solution to this difficulty [43]. Most recent claims about a solution based on anthropic considerations in the context of the *landscape* approach (see below) are not consensual.

Supersymmetry: After the pioneering work of Wess, Zumino, Deser, van Nieuwenhuizen, Freedman and others in the mid 1970's, a fundamental new symmetry has emerged from the drawing board of the theoreticians, *supersymmetry*. This symmetry relates bosons and fermions with equal masses and prevents that radiative corrections in the SM endow the Higgs field with a mass as high as the Planck mass. This suggests that new physics beyond the SM is needed in order to bridge the gap between the electroweak scale, $M_{EW} \simeq 10^2 \text{ GeV}$, and the quantum gravity scale. This is achieved as supersymmetry allows for a cancellation of the divergencies arising from bosons loops with the ones arising from fermions loops. If supersymmetry was an unbroken symmetry, this feature would explain the vanishing of the cosmological constant. This is clearly not the case, as known bosons and fermions do not have the same mass. This means that the spectrum of elementary particles must be much larger than the one that has been unraveled so far.

A striking feature of supersymmetry is that the algebra, *i.e.*, the anti-commutator, of its generators is proportional to the energy-momentum operator. Thus, a local version of supersymmetry corresponds to a general coordinate transformation and hence, gravity can be accommodated in a local supersymmetric theory. For this very reason, local supersymmetric theories are called supergravity theories (see Refs. [44, 45] for extensive reviews). Supergravity is an important step towards the unification of gravity with gauge theories.

A distinct property of supergravity is the presence of one (or more generally, $N \leq 8$) partner(s) of the graviton, the gravitino, a spin 3/2 particle that, through the super Higgs mechanism acquires, (in $N = 1$ supergravity) a mass [46] given by

$$m_{3/2} = \sqrt{\frac{8\pi}{3}} \frac{M_{SB}^2}{M_P}, \quad (54)$$

where M_{SB} is the supersymmetry breaking scale. Thus, if supersymmetry is broken at an intermediate scale between the electroweak scale and the typical GUT scale, $M_{SB} \simeq 10^{11} \text{ GeV}$, one should expect a signature of supergravity at the LHC collider — a quite exciting possibility.

Supersymmetry is also a fundamental ingredient of superstring theory, the most developed approach to understand in an unified fashion all interactions of Nature and to harmonize the description of gravity with quantum mechanics (see below).

As mentioned above, phenomenologically, supersymmetry is needed to ensure the rendezvous of coupling constants at about 10^{16} GeV [31]. Supersymmetry does also provide many candidates for the *dark matter* (see Refs. [47, 48] for recent reviews) of the Universe, given that it contains in its spectrum many neutral long lived weakly interacting massive particles (WIMPS); the linear combination of supersymmetric particles with the mentioned features is usually referred to as *neutralinos*.

Superstring/M-Theory Unification

The basic assumption of string theory is that the fundamental building blocks of reality are not particles, but rather extended one dimensional objects, *quantum strings*. These quantum strings can be open or closed, and particles correspond to the modes of excitation of the strings. Open strings ask for “branes” of any dimensionality not just *membranes*, a two dimensional object, to “support” their ends. A striking feature of superstring theory is that at these contact points the interactions correspond to those of supersymmetry gauge theories. On its hand, closed strings admit the graviton in its spectrum, besides a scalar particle, the dilaton. Thus, a rather model independent prediction of string theory is that the emerging effective gravity theories arising from it are scalar-tensor theories of gravity with higher-order curvature terms [49].

The obstacles encountered in the development of the Kaluza-Klein approach seemed, as discussed above, insurmountable till the “first string revolution” in 1984. Indeed, it was then shown by Green and Schwarz that higher order curvature terms do allow for non-trivial

gauge fields configurations after compactification of $D = 10$ down to 4 dimensions, but also that to ensure the mutual cancellation of gauge and gravitational anomalies the only GUT gauge groups admissible are $E_8 \otimes E_8$ or $SO(32)$ [50]. This breakthrough took place in the context of the supersymmetric string theory whose consistency (Lorentz symmetry and unitarity) requires $D = 10$ space-time dimensions (the bosonic string demands $D = 26$ [51]).

Actually, in its very first avatar, string theory was proposed to describe hadronic physics. However, the persistent appearance of massless vector and tensor states in its spectrum and the fact that $D > 4$ made the approach untenable for the description of hadrons. The first suggestion that string theory should instead be regarded as a unified theory of all interactions was put forward in 1974 by Scherk and Schwarz, based on the fact that the massless vector and tensor particles interact precisely as Yang-Mills gauge fields and the graviton [52]. The former feature could be achieved by assuming that the fundamental length scale of the theory $L \simeq \alpha'^{1/2} = T^{-1/2}$, where α' is the so-called Regge slope and T the string tension, should be identified with the Planck length, $L_P = \sqrt{G\hbar/c^3} = 1.6 \times 10^{-35} \text{ m}$.

The “first string revolution” did suggest a promising scenario to the understanding of our world. Starting from the $E_8 \otimes E_8$ 10-dimensional superstring theory, the so-called heterotic string theory [53], it is natural to demand that the dimensional reduction process down to 4 dimensions should preserve supersymmetry. This requirement turns out to be quite restrictive, as it demands that 6 extra space dimensions are compact, have a complex structure, no Ricci curvature and an $SO(3)$ holonomy group. That is, this compact space must be a Calabi-Yau manifold [54], a possibility that was thought to have opened the way to explain the origins of the SM.

A “second string revolution” has emerged from the discovery of the deep connection between all string theories. This is achieved through the so-called S and T dualities and the existence of an encompassing master theory, M -theory, which at low energies can be described by a $D = 11$, $N = 1$ supergravity theory [55]. The difficulty of M-theory is that it admits a huge number of solutions, about 10^{100} or greater — and every possible value for the cosmological constant and coupling constants. The space of all such string theory vacua is often referred to as the *landscape* [56].

In this context, a quite radical scenario emerges, namely that the multiple vacua of string theory is associated to a vast number of “pocket universes” in a single large *multiverse*. These pocket universes, like the expanding universe we observe around us, are all

beyond any observational capability, as they lie beyond the cosmological horizon [57]. The implications of these ideas are somewhat disturbing: the vacuum that corresponds to our Universe must arise from a selection procedure, to be dealt with via anthropic or quantum cosmological considerations. That is to say that our existence somehow plays a role in the selection process. If, from one hand, the vast number of vacua in the landscape ensures the reality of our existence, a selection process must be evoked. One refers to the *anthropic landscape*, when the vacuum selection is based on anthropic considerations. This interpretation is not free from criticism: indeed, it has been pointed out, for instance, that the impossibility of observing a multiverse implies that its scientific status is questionable — It is in the realm of metaphysics, rather than of physics [58]. This situation could be altered if the universes could interact. This possibility has been suggested in order to tackle the cosmological constant problem [59].

Unification in Cosmology

Cosmology is a particularly fruitful testing ground for unification ideas, given that the description of the Universe's history and evolution requires the integration of all physical knowledge. In the last few decades, the Hot Big Bang (HBB) model (see *e.g.* Ref. [60] for an extensive discussion, even though in Portuguese), which is based most fundamentally in general relativity and quantum field theory, nuclear physics, statistical mechanics, *etc.*, has acquired the status of a paradigm. Indeed, the HBB model harmonizes all known observational facts, provided one admits the existence of states beyond the SM: *dark energy*, which in its simplest form can be just a (fairly small) cosmological constant and *dark matter*. The HBB requires, at very early times, a period of accelerated expansion called *inflation* (see *e.g.* Ref. [61] for an extensive discussion).

Inflation reconciles cosmology with causality, solving the horizon, the homogeneity and the rotation problems. Inflation also suggests an elegant mechanism for structure formation, based on the ubiquitous quantum fluctuations that all fields are subjected to, and in particular, the scalar field responsible for inflation, the *inflaton*. In the context of GUTs, inflation can also prevent that magnetic monopoles dominate the dynamics of the Universe and lead to its collapse just after the Big Bang. Inflation naturally connects cosmology with GUTs, supergravity, superstrings and, in general, with the physics at the very early Universe (see *e.g.* Ref. [62]).

The recent discovery of the late acceleration of the Universe has lead to the necessity

of considering the existence of dark energy, and likewise inflation, a putative scalar field, dubbed *quintessence*, to drive the accelerated acceleration (see Ref. [63] for a review). It is interesting that some *quintessential inflationary models* have been suggested in order to unify inflation and dark energy [64]. The unification of dark energy and dark matter has also been proposed in the context of the Chaplygin equation of state and its generalization [65], with quite interesting phenomenological features.

III. THE FUTURE OF UNIFICATION

It is quite difficult, if not impossible, to predict the details of future developments in physics, and even harder to speculate about their inevitable turns and inflections. The unity of Nature and successes in describing the Universe along the lines of our discussion strongly suggest that the unification principle is a fruitful methodological tool. We have also seen that the lack of experimental evidence and phenomenological guidelines may render sterile any premature attempts of unification.

In this respect, the physics community is eagerly waiting for any new piece of information arising from the LHC, dark matter searches, SuperKamiokande-III, Planck Surveyor, GLAST/Fermi experiment, *etc.*, to direct its efforts. Naturally, in what concerns for instance the LHC, the main goals are the detection of the Higgs boson and of states related with supersymmetric extensions of the SM. Other objectives include the search for new forces of Nature, for states associated with the existence of extra dimensions and new states such as unparticles and ungravity, the latter associated to a putative infrared scale invariance of the SM [66].

On the theoretical front, it is evident that a deeper grasp of the strong coupling regime of field theories is required in order to address fundamental problems, from the existence of glue balls in QCD to the vacuum selection in string theory.

Another issue that deserves particular attention is whether the ultimate description of space-time requires more general algebraic structures such as, for instance, *noncommutative geometry* [67]. It is known that the position of strings on a D-brane satisfies the algebra of noncommutative geometry for a constant non-vanishing Kalb-Ramond field [68, 69], which suggests a connection between string theory and more complex algebraic structures, yet to be fully unraveled. An interesting related question concerns the possibility of extending noncommutative geometry to the phase space [21–23]. This seems to be a particularly rele-

vant issue, given that phase space noncommutative geometry has many interesting features at the level of quantum cosmology [70] . These include the possibility of obtaining quadratically integrable wave functions that solve the Wheeler-DeWitt equation for a Schwarzschild black hole and whose associated probability of reaching the singularity vanishes [71].

In the past, a rich lore of new phenomena was found while focusing on the implications of known fundamental symmetries and also on aspects and conditions of their violation. This might be particularly important, given that quantum field theory and general relativity rely fundamentally on Lorentz invariance - in its local version in the case of general relativity. In quantum field theory, the *breaking of Lorentz invariance* ensues the *breaking of CPT symmetry*. Given that the quantum field theories that describe the known interactions are local and unitary, one should not expect CPT violating effects. However, in the context of string field theory, solutions that violate Lorentz invariance and CPT symmetry have been found [72] and a great deal of research has been done in order to study the full range of phenomenological implications of these solutions [73] .

In what concerns general relativity, the most basic symmetry is the *Equivalence Principle*, which comprises the *Weak Equivalence Principle* (WEP), *Local Lorentz Invariance* (LLI) and *Local Position Invariance* (LPI). Despite the fact that these underlying invariances hold with great precision [4], it is quite possible that the Equivalence Principle is violated, most particularly by the dark sector (see Refs. [35, 74]). Clearly, any evidence about the breaking of the Equivalence Principle and any of its underlying assumptions may imply the crumbling of general relativity as we know it. Depending on the nature of the uncovered evidence, one can draw quite specific conclusions: for instance, the breaking of the WEP might indicate the existence of one or more new interactions of Nature at the particular range under scrutiny. The breaking of the LPI may point to a dependence of the coupling parameters on position and/or on time. These dependences might, for instance, be associated with the existence of extra dimensions (see *e.g.* Ref. [75] for a discussion and for an observational strategy for detection).

Finally, we mention the *Strong Equivalence Principle* (SEP), which states that gravitational self-energy couples to gravity likewise all other interactions. The validity of the SEP implies that spacetime geometry is uniquely determined by the metric, as prescribed by general relativity. Thus, further testing general relativity and its alternatives, such as for instance scalar-tensor theories of gravity and effective models arising from string theory, is a crucial line of future research — one that might open fruitful theoretical and observational

perspectives (see *e.g.* Refs. [76, 77] for discussions on space missions to test gravity).

Acknowledgments

We would like to thank the colleagues of the organizing committee, João Teixeira Pinto, Luís Saraiva, Pedro Girão, Pedro Gonçalves Henriques, Waldir Oliva e João Caraça, for the invitation to contribute in the homage to Aureliano Mira Fernandes, and Catarina Bastos and Jorge Páramos for the critical reading of the manuscript.

-
- [1] O. Bertolami, “The Mystical formula and the mystery of Khronos” in Minkowski Spacetime: A Hundred Years Later (Springer 2010) (<http://arxiv.org/abs/0801.3994>).
 - [2] O. Bertolami and F. Lobo, “Time and Causation”, *NeuroQuantol.* 7, 1 (2009) (<http://arXiv.org/abs/0902.0559>).
 - [3] O. Bertolami, “The Adventures of Spacetime”, in *Relativity and the Dimensionality of the World* (Springer 2007) (<http://arxiv.org/abs/gr-qc/0607006>).
 - [4] O. Bertolami, J. Páramos and S. G. Turyshev, “General theory of relativity: Will it survive the next decade?” in *Lasers, Clocks, and Drag-Free: Technologies for Future Exploration in Space and Tests of Gravity*; H. Dittus, C. Laemmerzahl, S. Turyshev, Eds. (Springer Verlag, 2006) (<http://arXiv.org/abs/gr-qc/0602016>).
 - [5] A. Einstein, “Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie”, *Sitzungsberichten der Preussischen Akad. d. Wissenschaften* (1917).
 - [6] S. Perlmutter et al., “Discovery of a supernova explosion at half the age of the universe”, *Nature* 391, 51 (1998);
S. Perlmutter et al., “Measurements of Omega and Lambda from 42 High-Redshift Supernovae”, *Ap. J.* 517, 565 (1999);
A. Riess et al., “Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant”, *Ap. J.* 116, 1009R (1998).
 - [7] H. F. M. Goenner, “On the history of unified field theories”, *Living Rev. Rel.* 7, 2 (2004) (<http://relativity.livingreviews.org/Articles/lrr-2004-2/>).
 - [8] H. F. M. Goenner, “Zur Geschishte der einheitlichen Feldtheorie: Einsteins Jahre erfolgloser Forschung”, *Ann. Phys. (Leipzig)* 15, 149 (2006).

- [9] A. Einstein, “The Meaning of Relativity: Four Lectures delivered at Princeton University”, (1950).
- [10] D. Z. Freedman, G. Gibbons and P. C. West, “Ten Into Four Won’t Go”, Phys. Lett. B 124, 491 (1983).
- [11] E. Witten, “Search for a Realistic Kaluza-Klein Theory”, Nucl. Phys. B 186, 412 (1981).
- [12] E. Witten, “Some Exact Multi-Instanton Solutions of Classical Yang-Mills Theory”, Phys. Rev. Lett., 38, 121 (1977);
P. Forgacs and N.S. Manton, “Space-Time Symmetries in Gauge Theories”, Comm. Math. Phys. 72, 15 (1980).
- [13] Yu.A. Kubyshin, J.M. Mourão, G. Rudolph and I.P. Volobujev, “Dimensional Reduction of Gauge Theories, Spontaneous Compactification and Model Building”, Lecture Notes in Physics 349 (Springer-Verlag, Berlin Heidelberg 1989).
- [14] O. Bertolami, J.M. Mourão, R.F. Picken and I.P. Volobujev, “Dynamics of euclidenized Einstein Yang-Mills systems with arbitrary gauge groups”, Int. J. Mod. Phys. A 6, 4149 (1991).
- [15] O. Bertolami, Yu.A. Kubyshin and J.M. Mourão, “Stability of compactification in Einstein Yang-Mills theories after inflation”, Phys. Rev. D 45, 3405 (1992).
- [16] Aureliano de Mira Fernandes, “Sulla teoria unitaria dello spazio fisico”, Rendiconti della Real Accademia dei Lincei, XV (1932).
- [17] Aureliano de Mira Fernandes, ‘Sulla teoria unitaria dello spazio fisico’, Rendiconti della Real Accademia dei Lincei, XVII (1933).
- [18] Aureliano de Mira Fernandes, “Modernas Concepções de Mecânica” (Imprensa da Universidade, Coimbra 1933).
- [19] L. A. Kurgan and P. Muzilek, “A survey of Knowledge Discovery and Data Mining process models”, The Knowledge Engineering Review, 21 (1), 1 (2006);
T. Hey and A. Trefehen, “e-Science and its implications”, Phil. Trans. R. Soc. Lond. A 361 (no. 1809) 1809 (2003).
- [20] P. A. M. Dirac, “Principles of Quantum Mechanics”, (Clarendon Press, Oxford 1930).
- [21] O. Bertolami, J. G. Rosa, C. Aragão, P. Castorina and D. Zappalà, “Noncommutative gravitational quantum well”, Phys. Rev. D 72, 025010 (2005).
- [22] O. Bertolami, J. G. Rosa, C. Aragão, P. Castorina and D. Zappalà, “Scaling of variables and the relation between noncommutative parameters in noncommutative quantum mechanics”,

- Mod. Phys. Lett. A 21, 795 (2006).
- [23] C. Bastos, O. Bertolami, N. Costa Dias and J. N. Prata, “Weyl-Wigner Formulation of Noncommutative Quantum Mechanics”, J. Math. Phys. 49, 072101 (2008).
 - [24] R. P. Feynman, “Quantum Electrodynamics” (Westview Press, 1998).
 - [25] V. B. Smocovitis, “Unifying Biology: The Evolutionary Synthesis and Evolutionary Biology” (Princeton University Press, Princeton 1996).
 - [26] W. J. Klous and R. I. Tilling, “The Dynamic Earth: The Story of Plate Tectonics”, (DIANE Publishing, 1996).
 - [27] W. Winter, “Lectures on neutrino phenomenology”, <http://arxiv.org/abs/1004.4160>.
 - [28] G. G. Ross, “Grand Unified Theories”, (Addison-Wesley Publishing Company, 1985).
 - [29] H. Georgi and S. Glashow, “Unity of All Elementary-Particle Forces”, Phys. Rev. Lett., 32, 438 (1974).
 - [30] H. Georgi, H. R. Quinn, and S. Weinberg, “Hierarchy of Interactions in Unified Gauge Theories”, Phys. Rev. Lett. 33, 451 (1974).
 - [31] U. Amaldi, W. de Boer and H. Fürstenau, “Comparison of grand unified theories with electroweak and strong coupling constants measured at LEP”, Phys. Lett. B 260, 447 (1991).
 - [32] A. D. Dolgov, “Baryogenesis, 30 Years After”, <http://lanl.arxiv.org/abs/hep-ph/9707419>.
 - [33] O. Bertolami, D. Colladay, V. A. Kostelecký and R. Potting, “CPT violation and baryogenesis”, Phys. Lett. B 395, 178 (1997).
 - [34] Ya. B. Zel’dovich, “The cosmological constant and the theory of elementary particles”, Sov. Phys. Usp., 11, 381, (1968).
 - [35] O. Bertolami, “The Cosmological constant problem: A User’s guide”, Int. J. Mod. Phys. D 18, 2303 (2009).
 - [36] A.D. Linde, “Is the Lee constant a cosmological constant?”, JETP Lett. 19, 183,(1974).
 - [37] J. Dreitlein, “Broken Symmetry and the Cosmological Constant”, Phys. Rev. Lett. 33, 1243 (1974).
 - [38] M. Veltman, “Cosmology and the Higgs Mechanism”, Phys. Rev. Lett. 34, 777 (1975).
 - [39] S. Weinberg, “The Cosmological Constant Problem”, Rev. Mod. Phys. 61, 1 (1989).
 - [40] S. Carroll, “The Cosmological Constant”, Living Rev. Rel. 4, 1 (2001).
 - [41] F. Bauer, “The Cosmological Constant and Discrete Space-Times”, <http://lanl.arxiv.org/abs/hep-th/0610178>.
 - [42] O. Bertolami, “Time Dependent Cosmological Term”, Il Nuovo Cimento 93, 36 (1986);

- O. Bertolami, “Brans-Dicke Cosmology With A Scalar Field Dependent Cosmological Term”, Fortsch. Phys. 34, 829 (1986).
- [43] E. Witten, “The Cosmological Constant from the Viewpoint of String Theory”, hep-ph/0002297.
- [44] P. van Nieuwenhuizen, “Supergravity”, Phys. Rep. 68, 189 (1981).
- [45] H. P. Nilles, “Supersymmetry, Supergravity and Particle Physics”, Phys. Rep. 110, 1 (1984).
- [46] S. Deser and B. Zumino, “Broken Supersymmetry and Supergravity”, Phys. Rev. Lett. 38, 1433 (1977).
- [47] J. Einasto, “Dark Matter”, <http://arXiv.org/abs/0901.0632>.
- [48] M. Roos, “Dark Matter: The evidence from astronomy, astrophysics and cosmology”, <http://arXiv.org/abs/1001.0316>.
- [49] B. Zwiebach, “Curvature Squared Terms and String Theories”, Phys. Lett. B 156, 315 (1985);
A. A. Tseytlin, “Ambiguity in the effective action in string theories”, Phys. Lett. B 176, 92 (1986);
D. Gross and E. Witten, “Superstring Modifications of Einstein’s Equations”, Nucl. Phys. B 277, 1 (1986);
M.C. Bento and N. E. Mavromatos, “Ambiguities in the Low-Energy Effective Actions of String Theories with the Inclusion of Antisymmetric Tensor and Dilaton Fields”, Phys. Lett. B 190, 105 (1987);
R. R. Metsaev and A. A. Tseytlin, “Curvature Cubed Terms In String Theory Effective Actions”, Phys. Lett. B 185, 52 (1987);
D. Gross and J. Sloan, “The Quartic Effective Action for the Heterotic String”, Nucl. Phys. B 291, 41 (1987);
M. C. Bento, O. Bertolami, A. B. Henriques and J. C. Romão, “Order α'^2 terms in the gravitational sector of string effective actions with the inclusion of the dilaton field”, Phys. Lett. B 218, 162 (1989).
- [50] M. B. Green and J. H. Schwarz, “Anomaly Cancellation in Supersymmetric $D = 10$ Gauge Theory and Superstring Theory”, Phys. Lett. B 149, 117 (1984).
- [51] A. M. Polyakov, “Quantum geometry of bosonic strings”, Phys. Lett. B 103, 207 (1981).
- [52] J. Scherk and J. H. Schwarz, “Dual Models for Nonhadrons”, Nucl. Phys. B 81, 118 (1974).
- [53] D. J. Gross, J. A. Harvey, E. J. Martinec and R. Rohm, “The Heterotic String”, Phys. Rev. Lett. 54, 502 (1985);

- D. J. Gross, J. A. Harvey, E. J. Martinec and R. Rohm, “Heterotic String Theory. 1. The Free Heterotic String”, Nucl. Phys. B 256, 253 (1985);
- D. J. Gross, J. A. Harvey, E. J. Martinec and R. Rohm, “Heterotic String Theory. 2. The Interacting Heterotic String”, Nucl. Phys. B 267, 75 (1986).
- [54] P. Candelas, G. Horowitz, A. Strominger and E. Witten, “Vacuum Configurations for Superstrings”, Nucl. Phys. B 258, 46 (1985).
- [55] E. Witten, “String theory dynamics in various dimensions”, Nucl. Phys. B 443, 85 (1995).
- [56] R. Bousso and J. Polchinski, “Quantization of four form fluxes and dynamical neutralization of the cosmological constant”, JHEP 0006, 006 (2000).
- [57] L. Susskind, “The Anthropic Landscape of String Theory”, <http://lanl.arxiv.org/abs/hep-th/0603249>;
- L. Susskind, “The cosmic landscape: String theory and the illusion of intelligent design” (Little, Brown, New York 2005).
- [58] G.F.R. Ellis, “On horizons and the cosmic landscape”, Gen. Rel. Grav. 38, 1209 (2006).
- [59] O. Bertolami, “A Curvature Principle for the interaction between universes”, Gen. Rel. Grav. 40, 1891 (2008).
- [60] Orfeu Bertolami, “O Livro das Escolhas Cósmicas” (Gradiva, Lisboa 2006).
- [61] K. Olive, “Inflation”, Phys. Rep. 190 (6), 307 (1990).
- [62] O. Bertolami, “Cosmological Implications of Supergravity and Superstring Theories”, Ph. D. thesis, University of Oxford, UK (1987).
- [63] E. J. Copeland, M. Sami and S. Tsujikawa, “Dynamics of dark energy”, Int. J. Mod. Phys. D 15, 1753 (2006).
- [64] P. J. Peebles and A. Vilenkin, “Quintessential inflation”, Phys. Rev. D 59, 063505 (1999);
- R. Rosenfeld and J. A. Frieman, “Cosmic microwave background and large-scale structure constraints on a simple quintessential inflation model”, Phys. Rev. D 75, 043513 (2007).
- O. Bertolami and V. Duvvuri, “Chaplygin inflation”, Phys. Lett. B 640, 121 (2006).
- [65] A. Yu. Kamenshchik, U. Moschella, V. Pasquier, “An alternative to quintessence”, Phys. Lett. B 511, 265 (2001);
- N. Bilić, G. B. Tupper and R. D. Viollier, “Unification of Dark Matter and Dark Energy: the Inhomogeneous Chaplygin Gas”, Phys. Lett. B 535, 17 (2002);
- M. C. Bento, O. Bertolami and A. A. Sen, “Generalized Chaplygin gas, accelerated expansion and dark-energy-matter unification”, Phys. Rev. D 66, 043507 (2002).

- [66] H. Georgi, “Unparticle Physics”, Phys. Rev. Lett. 98, 221601(2007).
- [67] A Connes, “Noncommutative geometry and physics”,
<http://www.alainconnes.org/docs/einsymp.pdf>
- [68] N. Seiberg and E. Witten, “String Theory and Noncommutative Geometry”, JHEP 9909, 032,(1999).
- [69] V. Schomerus, “D-branes and deformation quantization”, JHEP 9906, 030 (1999).
- [70] C. Bastos, O. Bertolami, N. Costa Dias and J. N. Prata, “Phase Space Noncommutative Quantum Cosmology”, Phys. Rev. D 78, 023516 (2008);
C. Bastos, O. Bertolami, N. Costa Dias and J. N. Prata, “Black Holes and Phase Space Noncommutativity”, Phys. Rev. D 80, 124038 (2009).
- [71] C. Bastos, O. Bertolami, N. Costa Dias and J. N. Prata, “The singularity problem and phase-space noncanonical noncommutativity”, <http://arxiv.org/abs/0912.4027>.
- [72] V. Alan Kostelecký and S. Samuel, “Spontaneous breaking of Lorentz symmetry in string theory”, Phys. Rev. D 39, 683 (1989);
V.A. Kostelecký and R. Potting, “Expectation Values, Lorentz Invariance and CPT in the Open Bosonic String”, Phys. Lett. B381, 89 (1996).
- [73] R. Lehnert, “CPT and Lorentz violation as signatures for Planck-scale physics”, <http://arxiv.org/abs/0907.1319>;
V. Alan Kostelecký, “Perspectives on Lorentz and CPT Violation”, <http://arXiv.org/pdf/0802.0581>;
O. Bertolami, “Threshold effects and Lorentz symmetry”, <http://arxiv.org/abs/hep-ph/0301191>;
O. Bertolami, “Ultra high-energy cosmic rays and symmetries of space-time”, Gen. Rel. Grav. 34,707 (2002).
- [74] O. Bertolami, F. Gil Pedro and M. Le Delliou, “Dark Energy-Dark Matter Interaction and the Violation of the Equivalence Principle from the Abell Cluster A586”, Phys. Lett. B 654, 165 (2007);
O. Bertolami, F. Gil Pedro and M. Le Delliou, “The Abell Cluster A586 and the Equivalence Principle”, Gen. Rel. Grav. 41, 2839 (2009).
- [75] S.G. Turyshev et al., “Advancing fundamental physics with the Laser Astrometric Test of Relativity”, Exper. Astron. 27, 27 (2009).
- [76] B. Christophe et al., “Odyssey: A Solar System Mission”, Exper. Astron. 23, 529 (2009).

- [77] P. Wolf et al., “Quantum Physics Exploring Gravity in the Outer Solar System: The Sagas Project”, *Exper. Astron.* 23, 651 (2009).